# From the center of pressure to the center of gravity, a new algorithm for a step forward in stabilometry 

# A partir do centro de pressão para o centro de gravidade, um algoritmo novo para um passo à frente na estabilometria 

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#### Abstract

Introduction: For more than thirty years, in clinical stabilometry, we have been using the center of pressure (CoP) to calculate stabilometric parameters. That was a mistake because the CoP signal comprises two series of information, one on the position of the center of gravity (CoG), and the other on the acceleration of the CoG. Objective: A step forward must be taken in order to separate these variables clearly using the CoG instead of the CoP to calculate stabilometric parameters. A lot of methods have been proposed to obtain the CoG from the CoP, yet none of them is used. We present a new algorithm for the same purpose. Method: A new mathematical way for solving the differential equation of DA Winter is proposed, which can use the "edge effects" due to known boundary conditions of the variables. Result: Solving the Winter's equation has two interests: Clinicians may think about what is observed through a model, and inter-subjects comparisons are better thanks to Winter's coefficient. Conclusion: During its next session, the international Committee for standardization of clinical stabilometry must choose one method to obtain the CoG from the CoP, before this choice is made, this new method must be known, well known and well understood because it could be the best choice.


Keywords: Clinical Stabilometry; Standardization; Center of Pressure; Center of Gravity.

## RESUMO

Introdução: Por mais de trinta anos, em estabilometria clínica, tem sido utilizado o centro de pressão (CP) para calcular parâmetros estabilométricos. Isso foi um erro, porque o sinal CdP compreende duas séries de informações, um sobre a posição do centro de gravidade (CG), e outro sobre a aceleração da roda denteada. Objetivo: Apresentar um novo algoritmo com a finalidade de separar estas variáveis usando o CG em vez da CP para calcular parâmetros estabilométricos. Vários métodos têm sido propostos para obter o CG do CP, mas nenhum deles é utilizado. Método: Uma nova forma matemática para resolver a equação diferencial da DA Winter é proposto, que pode usar os "efeitos de borda", devido à conhecidas condições de contorno das variáveis. Resultado: Resolver a equação de Winter tem dois interesses: Os médicos podem pensar sobre o que é observado através de um modelo, e as comparações inter-sujeitos são melhores graças ao coeficiente de Winter. Conclusão: Durante a sua próxima sessão, o Comité internacional para padronização de estabilometria clínica deve escolher um método para obter o CG da CP, antes que esta escolha é feita, este novo método deve ser conhecida, bem conhecida e compreendida, pois poderia ser a melhor escolha.
Palavras-chave: estabilometria clínica; Padronização; Centro de pressão; Centro de gravidade.

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## INTRODUCTION

Therapists, who practice stabilometry during their clinical examinations of functional disorders of the upright postural control system, use platforms that measure only the vertical forces exerted on the platform by the subject standing still. From these measurements, the current softwares calculate what is commonly called the CoP, i.e. the point of application of the resultant of the reaction forces exerted by the platform.

But this CoP is not the projection of the center of mass (CoM) on the plane of the platform. ${ }^{(1,2)}$ By equating the CoP to the projection of the CoM, a mistake is made that can be of importance. ${ }^{(3)}$ Moreover the mechanical study of the CoP shows that it mixes up two signals: the position of the CoG and its acceleration. The variations of the first are slow, the other is constituted by relatively short impulses of fairly high frequencies. ${ }^{(4,5)}$ These are indeed two quite different mechanical signals; calculate parameters straight from this sum could mean nothing from a biomechanical point of view. For instance, what does the speed of this signal mean? (figure 1).


Figure 1. Speeds of the COP and of the CoG, from the same stabilometric record of a rifle shooter, two seconds ( t ) before and after shooting.

This major defect of the CoP signal has long been known and many solutions have been tried to calculate the position of the CoG starting from the CoP signal. None of these methods are currently in use, the reasons why they are not used have to be discussed, but not in this paper. This paper only aims at explaining the principles and the general process of a new method that firstly deserves being known, well known, discussed and understood before its status inside the standardized clinical stabilometry can be discussed. A link will be given towards a web page where its algorithm, written in Octave language, can be downloaded free.

## METHOD

The suggested method uses a biomechanical model. The importance of using a model in clinical stabilometry must be discussed, but that will be done later. The model used here is the very well known inverted pendulum model. Assimilate the human body to a pendulum pivoting around its ankles allows one to write mechanical equations that relate the position of the CoP to the position of the CoG through couples acting on this pendulum. The method uses the equation proposed by Winter and Eng ${ }^{(6)}$, illustrated in figure 2.

The body weight, W, and the reaction force, R , opposed by the platform to the weight of the body are two equal forces, opposite, seldom aligned. They act on the pendulum respectively at the distance $G$ and $P$ from the ankle joint. The resulting moment of the couples WG and RP is equal


Figure 2. Diagram of Winter's mechanical equation. W: weight of the subject with its application point at the CoM. R: reaction force exerted by the platform. G : distance of the ankle axis to the vector W. P: distance of the ankle axis to the vector R. h: distance from the ankle axis to the CoM. $\alpha$ : angle between the pendulum and the vertical
to the moment of inertia of the pendulum, multiplied by its angular acceleration, $\alpha^{\prime \prime}$ :

$$
\begin{equation*}
W G-P R=I \alpha^{\prime \prime} \tag{1}
\end{equation*}
$$

The oscillations of the human pendulum being of low amplitude at rest, the angle $\alpha$ is not very different from its sinus in these conditions, so the angular acceleration, $\alpha^{\prime \prime}$, of the pendulum is almost equal to the horizontal acceleration of the center of gravity, $\mathrm{G}^{\prime \prime}$, divided by the distance, h , between the axis of the ankle and the CoM:

$$
\begin{equation*}
\alpha^{\prime \prime}=G^{\prime \prime} / h \tag{2}
\end{equation*}
$$

furthermore:

$$
\begin{equation*}
R=G=m g \tag{3}
\end{equation*}
$$

where $m$ is the mass of the subject, $g$ : the acceleration of gravity.

So the equation [1] can be written as:

$$
\begin{equation*}
G-P=\frac{I}{m g h} G^{\prime \prime} \tag{4}
\end{equation*}
$$

If we write

$$
\begin{equation*}
\frac{I}{m g h}=k^{2} \tag{5}
\end{equation*}
$$

then the equation [4] becomes :

$$
\begin{equation*}
P=G-k^{2} G^{\prime \prime} \tag{6}
\end{equation*}
$$

You need only to solve this differential equation for any sampled positions of the CoP, $\mathrm{P}_{\mathrm{i}}$, to determine the position of the CoG, $\mathrm{G}_{\mathrm{j}}$, at this moment, t , of the recording. Unfortunately, the equation [6] has an infinite number of solutions, $\nu_{i}$, because we know neither the initial position nor the initial speed of $G$ (even if you know the value of the acceleration of a mobile at every millisecond, you cannot know where the mobile at an instant $t$ is if you do not know where it started from, and which its initial speed was).

But a simple subtraction can solve this problem, because the difference between any two of the solutions, $\gamma_{i}$ and $\gamma_{\mathrm{k}^{\prime}}$ for the same $P_{j}$ is singular. Indeed, suppose «d» is the name of the difference between two solutions, $\gamma_{1}$ and $\gamma_{2}$, of the equation:

$$
\begin{equation*}
\gamma_{1}-\gamma_{2}=d \tag{7}
\end{equation*}
$$

According to the equation [6], this difference can be written:

$$
\begin{equation*}
P_{j}-P_{j}=\gamma_{1}-\gamma_{2}-k^{2}\left(\gamma_{1}^{\prime \prime}-\gamma_{2}^{\prime \prime}\right) \tag{8}
\end{equation*}
$$

or, by replacing $\gamma_{1}-\gamma_{2}$ by $d$ :

$$
d=k^{2} d^{\prime \prime}
$$

The equation [9] describes a function, $d$, which is equal to its second derivative $d^{\prime \prime}$. This function is well known and can be written this way:

$$
\begin{equation*}
d=G_{0} e^{-n t}+G_{F} e^{-n(F-t)} \tag{10}
\end{equation*}
$$

t : time variable; F : final time; $\mathrm{G}_{0}$ : initial solution; $\mathrm{G}_{\mathrm{F}}$ : final solution; $n=1 / k$.
$G_{0}$ and $G_{F}$ are not known, but it is hypothesized that any $G_{i}$ remains inside the support basis, the subject is standing upright quiet. The calculation of the value of $d$ with different possible values of $G_{0}$ or of $G_{F}$ - inside the support basis shows that $d$ is almost null after three seconds, whatever the value chosen for $G_{0}$ or $G_{F}$ be (fig. $3 \& 4$ ).

So, except during the first three seconds and the last three ones, all the solutions of the equation [6] are almost equal. In this interval, any solution of the differential equation for each $P_{j}$, is appropriate.


Figure 3. Evolution over time, from zero to three seconds, of the value of d according to an arbitrary selection of Go or GF, from 100 mm to 10 mm .


Figure 4. Zoom on the figure 3, from two to four seconds.

The equation [6] can be expressed best at any moment, $j$, of measurement by a linear equation, replacing the second derivative with a finite difference approximation.

For j from 1 to $n$, we can write a system of $n$ linear equations with $n$ unknowns, having a solution, $\mathrm{G}_{\mathrm{i}}$, and only one, assuming $G_{0}$ and $G_{n+1}$ are zero. An algorithm to solve this system of $n$ linear equations can be found free at this URL ${ }^{(7)}$, it is written in the Octave language (Compatible with Matlab) and it is called «SpG_N».

$$
\begin{equation*}
P_{j}=G_{j}-k^{2} \frac{G_{j-I}+G_{j+1}-2 G_{j}}{\delta t^{2}} \tag{11}
\end{equation*}
$$

## RESULTS

Comparisons have been made between the position of the CoG calculated by this mechanical method and the positions of the CoM measured by an optical mean, the two curves are very close (figure 5), see companion paper. ${ }^{(8)}$

The interest of working with the center of gravity appears obviously from recordings of marksmen while shooting, it is possible to compare the movements of their CdG (position, speed and acceleration) with the movements of the rifle and that led to interesting conclusions for shooters. ${ }^{(9)}$

## DISCUSSION

The method uses the inverted pendulum model, which is not perfect, but reasonable/satisfactory ${ }^{(10,11)}$, moreover, in the field of clinical posturology, we need a model in order to think about the biomechanical problems of the patient.

This method must be used only for subjects standing upright quiet, for two reasons: they must stand as much as possible as an inverted pendulum, and their CoP must be always inside the support basis due to the hypothesis of the calculation.

The experiments of Houel and Stubbe ${ }^{(8)}$ show that in the frontal plane - where the model is good - the method brings about an «excellent» result, whereas in the sagittal plane - where possible movements of the trunk, the arms, the head perturb the inverted pendulum - the result is only «very good».

In the equation [11] the value of $\mathrm{G}^{\prime \prime}$ is replaced by a finite difference approximation. The importance of the error introduced by this approximation was studied according to the sampling frequency. Theoretically the more the frequency increases, the better the accuracy of G" obtained by this calculation is. In fact it can be noticed that between 40 and 300 Hz , the result of the calculation of the position, velocity and acceleration of the CoG varies only just a thousandths (Fig. 6).

So, from a sampling frequency of 40 Hz ., the inaccuracy of the estimation of $\mathrm{G}^{\prime \prime}$ is meaningless.

Another advantage of the method lies in the " $k$ " factor that can be studied in order to improve the taking into account


Figure 5. Left-Right postural sway of the center of gravity (blue line) measured by the mechanical method and of the centre of mass (red line) measured by an optical method. (From L Stubbe and N Houel [8] by courtesy).


Figure 6. Effect of the sampling frequency on the precision of the method. Abscissa: sampling frequency. Ordinate: rapport between the mean value of the parameter and its value at a particular sampling frequency.
of the anthropometric features of the subject and, doing so, being able to improve the comparisons of the functioning of the upright postural control system between subjects of different heights and coefficients of form. ${ }^{(12)}$

## CONCLUSIONS

This new method enables to calculate the position of the CoG starting from the vertical forces measured by a force platform, using the model of the inverted pendulum and integrating the anthropometric characteristics of the subjects.

That represents a great step forward in the field of clinical stabilometry for two reasons:

- The stabilometric parameters can be calculated directly from the CoG, which is a great step as the parameters calculated from the CoP - as was common practice -
have no biomechanical meaning, since the CoP signal, mixes up two quite different mechanical data: the position of the CoG and the acceleration of the CoM.
- These new stabilometric parameters, that take into count the anthropometric characteristics of the subjects, allow to edit more precise reference values because they are no longer affected by the difference between subjects.


## AUTHORS CONTRIBUTION

BG mathematician, $O B$ is documentalist, PMG posturologist.

## COMPETING INTERESTS

The authors declare no conflicts of interest.

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